MAC 1105
Sullivan
Chapter 6 Exam (V2)
Sections 6.5 - 6.8

Name: _____KEY_____
Date: _____
Section: _____

Take Home Exam

You MUST show your work to receive full credit. This exam is worth 100 points. There are 6 built in bonus points. Good Luck!

Solve each equation: (6 points each)

1. $4^{1-2x} = 32$

$$2^{2(1-2x)} = 2^5$$
$$2^{2-4x} = 2^5$$
$$2 - 4x = 5$$
$$-4x = 3$$
$$\boxed{x = -3/4}$$

2. $25^{2x} = 5^{x^2-12}$

$$5^{2(2x)} = 5^{x^2-12}$$
$$4x = x^2 - 12$$
$$0 = x^2 - 4x - 12$$
$$0 = (x-6)(x+2)$$
$$\boxed{x = 6} \qquad \boxed{x = -2}$$

3. $\log_x 64 = -3$

$$64 = x^{-3}$$
$$4^3 = x^{-3}$$
$$\left(\tfrac{1}{4}\right)^{-3} = x^{-3}$$
$$\boxed{\tfrac{1}{4} = x}$$

4. $\log_3 \sqrt{x-2} = 3$

$$\sqrt{x-2} = 3^3$$
$$\sqrt{x-2} = 27$$
$$x - 2 = (27)^2$$
$$x = (27)^2 + 2$$
$$x = 729 + 2$$
$$\boxed{731}$$

5. $3e^{2x} = 5$

$$e^{2x} = \frac{5}{3}$$
$$\ln e^{2x} = \ln\left(\frac{5}{3}\right)$$
$$2x(\ln e) = \ln\left(\frac{5}{3}\right)$$
$$\boxed{x = \tfrac{1}{2}\ln\left(\frac{5}{3}\right)}$$
$$\boxed{x \approx 0.255}$$

6. $5^{x+2} = 3^{4x-2}$

$$\ln 5^{x+2} = \ln 3^{4x-2}$$
$$(x+2)\ln 5 = (4x-2)\ln 3$$
$$x\ln 5 + 2\ln 5 = 4x\ln 3 - 2\ln 3$$
$$x\ln 5 - 4x\ln 3 = -2\ln 5 - 2\ln 3$$
$$4x\ln 3 - x\ln 5 = 2\ln 5 + 2\ln 3$$
$$x(4\ln 3 - \ln 5) = \frac{(2\ln 5 + 2\ln 3)}{(4\ln 3 - \ln 5)}$$
$$\boxed{\approx 1.94}$$

7. $\log_6(x+3) + \log_6(x+4) = 1$

$\log_6(x+3)(x+4) = 1$

$(x+3)(x+4) = 6^1 = 6$

$x^2 + 4x + 3x + 12 = 6$

$x^2 + 7x + 6 = 0$

$(x+6)(x+1) = 0$

~~$x = -6$~~ $x = -1$

8. $\log x = 4.5$

$\log_{10} x = 4.5$

$x = 10^{4.5}$

$x = 31,622.78$

9. A child's grandparents purchase a \$16,000 bond that matures in 16 years to be used for her education. The bond pays 6% interest compounded semiannually. How much will the bond be worth at maturity?   **(8 points)**

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = (16,000)\left(1 + \frac{0.06}{2}\right)^{2(16)}$$

$$A = (16,000)(1.03)^{32} = \$41,201.32$$

10. The bones of a prehistoric man found in the desert of New Mexico contain approximately 5% of the original amount of carbon-14. If the half life of carbon-14 is 5600 years, approximately how long ago did the man die?   **(8 points)**

$P = P_o e^{kt}$

$\tfrac{1}{2}P_o = P_o e^{k(5600)}$

$0.5 = e^{5600k}$

$\ln 0.5 = \ln e^{5600k}$

$\ln 0.5 = (5600)k \cdot \ln e$

$\dfrac{\ln 0.5}{5600} = k = -1.238 \times 10^{-4}$

$0.05 P_o = P_o e^{(-1.238 \times 10^{-4})t}$

$0.05 = e^{(-1.238 \times 10^{-4})t}$

$\ln 0.05 = \ln e^{(-1.238 \times 10^{-4})t}$

$\ln 0.05 = (-1.238 \times 10^{-4})t \cdot \ln e$

$\dfrac{\ln 0.05}{(-1.234 \times 10^{-4})} = t$

$24,198$ years

11. If Kenya has a population of 20,000,000 people and doubling time of 16 years and if the growth continues at the same rate, find the population in 22 years. (8 points)

$$P = P_0 e^{kt}$$

$$40 = 20 e^{k(16)}$$

$$2 = e^{16k}$$

$$\ln 2 = \ln e^{16k}$$

$$\ln 2 = (16k) \ln e$$

$$\frac{\ln 2}{16} = k$$

$$0.0433 = k$$

$$P = P_0 e^{0.0433t}$$

$$P = (20,000,000) e^{0.0433(22)}$$

$$\boxed{P = 51,848,825 \text{ people}}$$

Write each expression as a single logarithm   (4 points each)

12. $4\log_2 x - 2\log_2 y - \dfrac{1}{2}\log_2 z$

$$\log_2 x^4 - \log_2 y^2 - \log_2 \sqrt{z}$$

$$\boxed{\log_2 \frac{x^4}{y^2\sqrt{z}}}$$

13. $\ln(x+2) - 7\ln(x) + 3\ln(x+7)$

$$\ln(x+2) - \ln x^7 + \ln(x+7)^3$$

$$\boxed{\ln \frac{(x+2)(x+7)^3}{x^7}}$$

Evaluate each expression (without using a calculator).   (4 points each)

14. $\log_2 \dfrac{1}{8} = \boxed{-3}$

$$\log_2 2^{-3}$$

15. $\log_3 81 = \boxed{4}$

$$\log_3 3^4$$

16. $\log_2 2^x = \boxed{x}$

$$x(\log_2 2)$$

17. $e^{3\ln x} = e^{\ln x^3} = \boxed{x^3}$

Evaluate each expression using a calculator. Round your answers to the nearest thousandth.

(2 points each)

18. $\log(5.972) =$ 0.776

19. $\log(-4.875) =$ DNE

20. $\ln(125) =$ 4.828

21. $\log_6 50 = \dfrac{\log 50}{\log 6} =$ 2.183

22. $\dfrac{\log 16}{\log 8} =$ 1.333